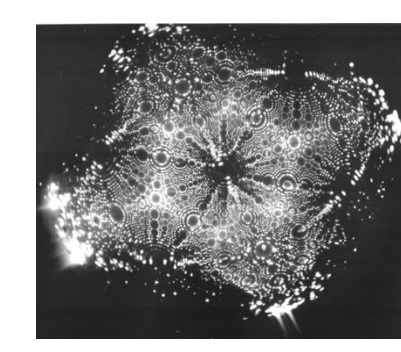
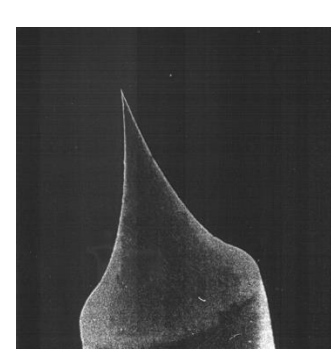




ON THE ADVANTAGES OF USING THE SCALED FIELD f , RATHER THAN THE NORDHEIM PARAMETER y , AS THE AUXILIARY PARAMETER IN MURPHY-GOOD FIELD ELECTRON EMISSION THEORY

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In the Murphy-Good field electron emission (FE) equation, the exponent contains a mathematical correction factor " v_F " that is a function of both the local work function and the local surface field. This factor can be expressed as a function of a single modelling variable in two alternative ways. The older method, developed in 1953, uses the Nordheim parameter y ; the more recent **method, developed in the period 2006 to 2010, uses the scaled field f , which is equal to y^2 . Both methods yield the same numerical results in contexts such as Fowler-Nordheim-plot analysis, and both are currently acceptable. However, it has been argued that the modern approach is mathematically superior and more suited to the further development of FE data analysis. This Poster aims to present arguments to back up these claims.**

1. INTRODUCTION

This Poster concerns the **Murphy-Good (MG) zero-temperature field electron emission (FE) equation**, first formulated in 1956 [1]. MG FE theory relates **local emission current density (LECD) J^{MG0}** to **local work function ϕ** and the **magnitude F** of the local surface electrostatic (ES) field by the equation

$$J^{MG0} = t_F^{-2} a \phi^{-1} F^2 \exp[-v_F b \phi^{3/2} / F], \quad (1)$$

where a and b are the Fowler-Nordheim universal constants and v_F and t_F are mathematical correction factors.

Nowadays, there are two forms of "modelling mathematics" for these correction factors. It can utilise: **either** the 1953 work of **Burgess, Kroemer and Houston (BKH)** [2], which uses the **Nordheim parameter y** as the independent variable; **or** the work of **Forbes and Deane (FD)** in the period 2006–2010 (e.g., see [3]). The FD method uses, as the independent variable, the **scaled field f** for a **Schottky-Nordheim (SN) barrier of zero-field height ϕ** .

This scaled field f is defined by

$$f \equiv c^2 \phi^{-2} F = (e^3 / 4\pi\epsilon_0) \phi^{-2} F, \quad (2)$$

where: c is the Schottky constant; e is the elementary (positive) charge, and ϵ_0 is the electric constant. The Nordheim parameter $y \equiv +f^{1/2}$.

Both approaches lead to the same numerical results in traditional applications of MG FE theory, and both are currently acceptable. However, the present author regards the FD approach as mathematically superior and better suited to the further development of FE data analysis. These issues have been partly discussed previously [3,4]. This note brings together current reasons for regarding the more modern "21st century" approach as superior.

2. SOME PURE MATHEMATICAL BACKGROUND

In FE theory it is useful to distinguish (a) "pure mathematics" and (b) "the mathematics of modelling". Also, it seems the best way of dealing with FE mathematics is to introduce a new special mathematical function (SMF) of mathematical physics, denoted here by $v_{FD}(x)$. $v_{FD}(x)$ is a special solution of the **Euler/Gauss Hypergeometric Differential equation (HDE)**. x is an abstract mathematical variable, called here the **Gauss variable** because it can be identified with the independent variable in the Gauss HDE.

The relevance of $v_{FD}(x)$ is that it appears in the solution of integrals of type:

$$\int M^{1/2}(z) dz \equiv \int (a_0 - a_1 z - a_{-1} z^{-1})^{1/2} dz, \quad (3)$$

where a_0 , a_1 and a_{-1} are positive constants and the integral is taken between two adjacent zeros of $M(z)$, across a region where $M(z)$ is positive. Integrals of this type are known to emerge in theories of: FE from free-electron metals; FE from semiconductors; field desorption; field evaporation; and one-dimensional theories of electrostatic field ionization. Thus, the SMF $v_{FD}(x)$ has several prospective applications.

Because $v_{FD}(x)$ is a solution of the Gauss HDE, it has many equivalent exact definitions. One of these uses **two infinite series**: the first few terms are:

$$v(x) = 1 - \left(\frac{9}{8} \ln 2 + \frac{3}{16}\right)x - \left(\frac{27}{256} \ln 2 - \frac{51}{1024}\right)x^2 - \left(\frac{315}{8192} \ln 2 - \frac{177}{8192}\right)x^3 + O(x^4) \\ + x \ln x \left[\frac{3}{16} + \frac{9}{512}x + \frac{105}{16384}x^2 + O(x^3) \right]. \quad (4)$$

This definition is applied to FE by setting $x=f$. **Because there are no terms in $x^{1/2}$ in eq. (4), there is no merit in setting $x=y^2$.**

An alternative definition is based on a **defining equation**:

$$x(1-x) d^2W/dx^2 - (3/16)W = 0. \quad (5)$$

$v_{FD}(x)$ is the solution of (5) that satisfies the (unusual) boundary conditions:

$$v_{FD}(0) = 1; \quad \lim_{x \rightarrow 0} \{dv_{FD}/dx - (3/16)\ln x\} = - (9/8)\ln 2. \quad (6)$$

If the defining equation is reformulated in terms of $y = x^{1/2}$, then it becomes

$$y(1-y^2) d^2W/dy^2 - (1-y^2) dW/dy - (3/4)yW = 0. \quad (7)$$

This y -based form (7) is significantly more complicated than the x -based form (5), and is not obviously a special case of the Euler/Gauss HDE.

3. SOME MINOR MATHEMATICAL ARGUMENTS

* Several traditional approximations for the correction factor v_F have the form

$$v_F \approx c_1 + c_2 y^2, \quad (8)$$

where c_1 and c_2 are numerical constants. There is **no obvious merit in preferring this form** over the simpler form:

$$v_F \approx c_1 + c_2 f. \quad (9)$$

* Since $df = 2ydy$, any expression involving dv or d^2v will be slightly more complicated when y is used, rather than f .

4. SOME PRACTICAL CONSIDERATIONS

Other reasons for preferring the use of f and $v_{FD}(f)$, rather than y and $v_{BKH}(y)$ include the following.

* The concept of "scaled field" is probably easier to understand than the concept of the Nordheim parameter (which is "scaled reduction in SN barrier height").

* The parameter f is well defined, whereas y sometimes relates (as here) to a potential-energy (PE) barrier of zero-field height equal to the local work function, sometimes to a barrier of arbitrary zero-field height.

* The parameter f is proportional to the local surface field-magnitude F , whereas y is proportional to $\sqrt{F}$. This often makes f easier to use, e.g. in the orthodoxy test [5].

* f is a useful parameter in other contexts: discussion of the FE characteristics of emitter materials of different work-functions is more uniform when carried out in terms of f rather than in terms of the local field-magnitude F . A particular example of this is when defining the temperature-field regime of applicability of the Murphy-Good finite-temperature formula for "cold field electron emission" (aka "deep-tunnelling emission").

* The Murphy-Good (MG) plot [6] is "more nearly straight" than a Fowler-Nordheim plot, and the extraction of emission area using a MG plot presents fewer difficulties than a FN plot does. It is easier to formulate the theory of a MG plot by using f than it would be to formulate it using y .

* The scaled field f can be defined more generally by the equation $f \equiv F/F_R$ where the reference field F_R is the field-magnitude needed to pull the top of the tunnelling barrier down to the Fermi level. The formulas for f given earlier apply specifically to the SN barrier, but the definition in terms of a reference field can be generalized to other forms of barrier. By contrast, the definition of the Nordheim parameter can be easily applied only to the SN barrier.

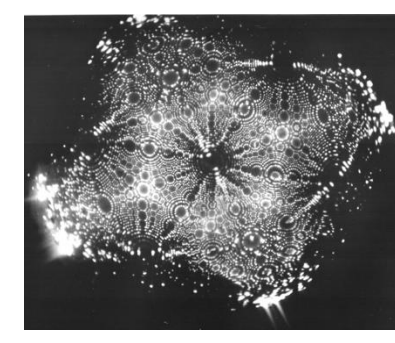
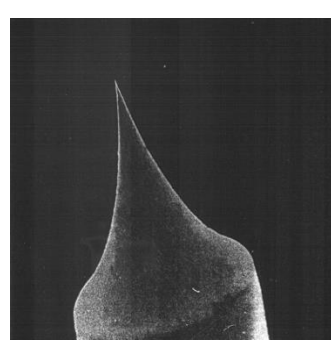
5. CONCLUSION

When implementing theory based on the Murphy-Good FE equation, this note has listed multiple reasons for preferring the use of the scaled field f and the function $v_{FD}(f)$, rather than the Nordheim parameter y and the function $v_{BKH}(y)$.

In the author's view, it would make for a simpler and clearer system if use of the Nordheim parameter y were gradually phased out, and relevant formulae and tables that currently use y were replaced by their equivalents that use either the Gauss variable x or the scaled field f .

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